Forty Years of Illustrating Mathematical Results

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Western University, Canada

Joint work with so very many people
These slides available at rcorless.github.io

Announcing Maple Transactions

an open access journal with no page charges or other fees

mapletransactions.org

Another announcement

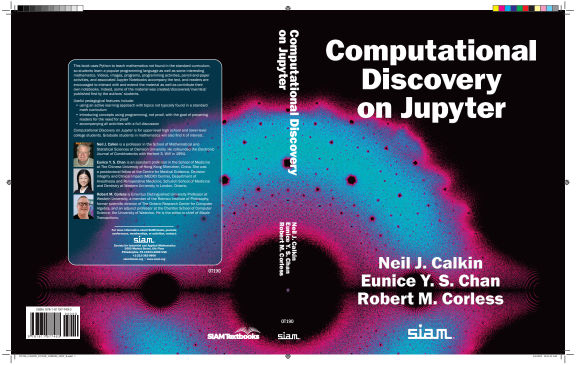

Figure 1: A 2023 book from SIAM: Calkin, Chan, & Corless, "Computational Discovery on Jupyter"

Yet another announcement

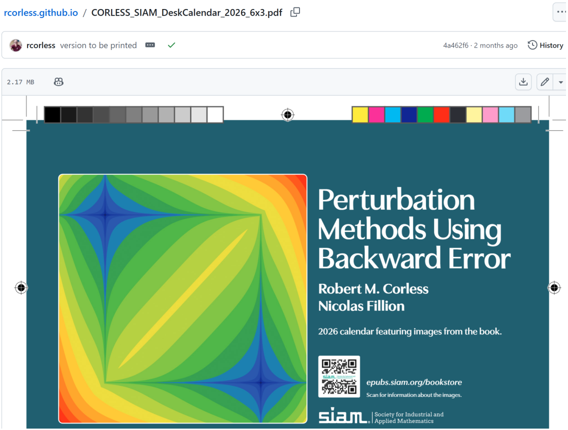

Figure 2: A 2026 book available April from SIAM: Corless & Fillion "Perturbation Methods Using Backward Error", and a calendar with images from the book

About me

- BSc 1980 UBC Math & Computer Science; MMath 1982 Waterloo Applied Math; PhD 1986 UBC Mech Eng*.
- University of Western Ontario Applied Math 1987–2019 (tenure 1993, full 1998, “Distinguished” 2006, Emeritus 2019)
- Editor-in-Chief Maple Transactions 2021–present
- Semantic Scholar says I have 230 publications. Could be true.

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* When my advisor, G.V. Parkinson, asked why I wanted to take a graduate course in number theory from David Boyd (on Pisot and Salem numbers), I started to explain; he cut me off, saying “Oh, interest. That’s fine.” and signed the form.

The purpose of illustration

For me (and probably for you, too) the purpose of illustration is to help tell the *story* of the mathematics being investigated.

Of course, that is also the purpose of every *formula* that we use, or indeed any other bit of the writing.

Early days for me

Some of my earliest published visualizations/illustrations were *hand-drawn*. I've used drafting tools, pen-plotters, film photography of computer screens, . . . The present-day tools are better. Well, for me, anyway.

As against that, my **favourite mathematical illustrations** actually are hand-drawn, but of course they are not mine: those of Fiona Ross and William Ross from their stunning paper "The Jordan Curve Theorem is Non-Trivial." If you don't know that paper, you should.

Some other hand-drawn illustrations

- Baker & Rippon 1985 a hand-drawn fractal. My version by computer which I think illustrates Carlsson's 1907 theorem better (I have brought some posters to give away)
- I was sure there was one by William J. Gilbert but I can't find it now; Gilbert & Vrscay did a paper on Schröder iteration that is relevant to a picture I constructed, which we re-purposed for the introduction to CDJ

A closer look

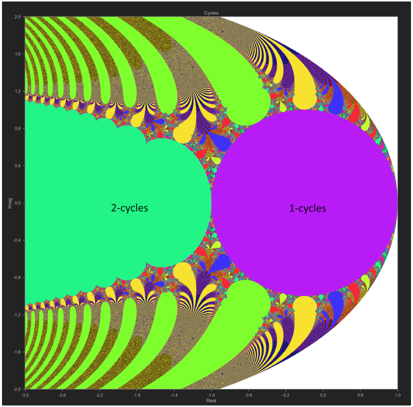

Figure 3: The fractal tower from this paper (2023)

Basic Rule

I agree with Jon Borwein: visualizations and illustrations should be faithful to the underlying mathematics. Artistic license is fine, but not at the expense of the mathematics. It is, after all, there to help tell the story.

Some more favourites from my own work

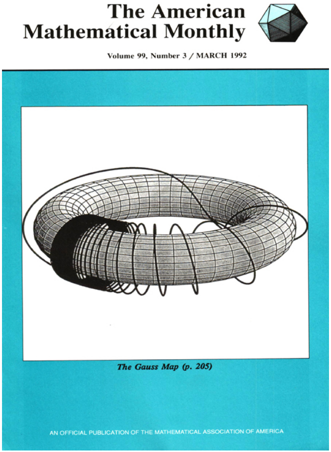

Figure 4: Cover of the March 1992 American Mathematical Monthly

That was hard to draw in 1991

- I wrote a *PostScript program* and had the *printer* do the computation for the flat graph in Fig. 1 in the paper (image was otherwise too big for the printer's memory!)
- The idea of drawing the curve on a torus was mine, and Fig. 2 in the paper was done in Maple (with help from George Labahn); but the cover image version of it was done in Mathematica by someone else (I never found out who)
- This is now an activity in the 2023 book Computational Discovery on Jupyter
- Hey, this was actually a number theory paper! "Continued fractions and chaos"

What did those images illustrate?

- That the apparent discontinuities of the Gauss map $G(x) = \mathrm{frac}(1/x)$ at $x = 1/n$ for $n = 1, 2, \ldots$ were "removable"
- That the singularity at $x = 0$ was quite important. This showed up in the *visualization process* as well because the curve just "blacks out" near $x = 0$ (fractal dimension 2, or nearly) (cf the Ross & Ross Jordan Curve paper!)
- finite-arithmetic versions ($x \in \mathbb{F}$ instead of $x \in \mathbb{R}$) behave *very* differently

A recent thread: Mandelbrot polynomials and Matrices

Here, illustrations frequently lead to questions:

1. Piers Lawrence & RMC, The Largest Root of the Mandelbrot Polynomials (Jonfest proceedings, 2013)
2. Neil J Calkin, Eunice Chan, & RMC, Some Facts and Conjectures about Mandelbrot Polynomials (Maple Transactions 2021)
3. Neil Calkin et al, A Fractal Eigenvector (American Math Monthly 2022) I will talk more about this, time permitting

Piers Lawrence (1987–2025) had the fundamental idea which opened the door to these results.

Another cover image

Figure 5: cover image: London Mathematical Society Newsletter, November 2020, page 16 (RMC, NJ Higham, & SE Thornton)

More about Bohemian matrices

Bohemian Matrices

1. Upper H–berg and Toeplitz Bohemians (Chan et al, 2020, LAA)
2. What can we learn from Bohemian matrices? (RMC, 2021 Maple Transactions)
3. Skew-symmetric tridiagonal Bohemian matrices (RMC 2021 Maple Transactions)
4. Bohemian Matrix Art in the Maple Primes Art Gallery
5. Bohemian Matrix Geometry ISSAC 2022 (Lille) The illustrations provoked a theorem that we were able to prove

A case study in a 3rd thread: Balanced Blends

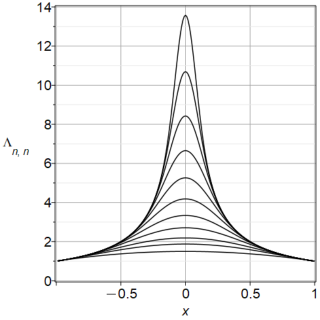

Figure 6: The Lebesgue function of balanced blends on [−1,1] grows like $2\sqrt{n/\pi}$ where the grade is $2n+1$. We plot the function for $n = 1$, 2, 3, 5, 8, 13, 21, 34, 55, 89, and 144. Lebesgue constants *must* be unbounded on [−1,1]; optimal is $2\ln(2n+1)/\pi + O(1)$, like Lagrange interpolation on Chebyshev nodes. But $\sqrt{n}$ is pretty decent. For $n = 144$ this is 13.5 vs 3.6. See also *https://www.chebfun.org/examples/approx/LebesgueConst.html*

Unbalanced Blends

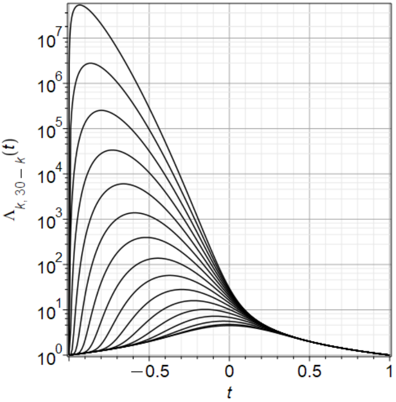

Figure 7: Unbalanced blends are exponentially bad

What's a Blend?

Suppose that we know some Taylor coefficients of a function at two distinct points, say $z = a$ and $z = b$. Then put $z = a + s(b - a)$ and the interval $0 \le s \le 1$ determines a line segment in the z-plane.

Then (Hermite, Cours d'Analyse 1873)

$$H(s) = \sum_{j=0}^{m} p_j \sum_{k=0}^{m-j} \binom{n+k}{k} s^{k+j} (1-s)^{n+1} + \sum_{j=0}^{n} (-1)^j q_j \sum_{k=0}^{n-j} \binom{m+k}{k} s^{m+1} (1-s)^{k+j} \tag{1}$$

has $H^{(j)}(0)/j! = p_j$ for $0 \le j \le m$ and $H^{(j)}(1)/j! = q_j$ for $0 \le j \le n$. Here differentiation is wrt s so one has to be careful about bookkeeping.

This is a "two-point Hermite interpolational polynomial," or "blend" for short, because it blends the Taylor series at either end together to give an approximant on the interval between.

On $0 \leq s \leq 1$ the Lebesgue constant is 2

The Lebesgue *function*—which the Lebesgue constant is a bound for—gives a bound for the condition number. For blends the Lebesgue function is exactly the polynomial you get with all series coefficients 1 at the left and all coefficients $(-1)^j$ on the right:

$$
\begin{aligned}
L_{m,n}(s) = & \sum_{j=0}^{m}\sum_{k=0}^{m-j}\binom{n+k}{k} s^{k+j}\,(1-s)^{n+1} \\
& + \sum_{j=0}^{n}\sum_{k=0}^{n-j}\binom{m+k}{k} s^{m+1}\,(1-s)^{k+j}
\end{aligned}
\tag{2}
$$

We can show that on $0 \leq s \leq 1$, $L_{m,m}(s) < 2$ and indeed $L_{m,m}(s) \leq L_{m,m}(1/2) = 2 - 2^{-2m+1}\binom{2m+2}{m+1} \sim 2 - 2/\sqrt{\pi m} + O(1/m^{3/2})$.

Bernstein polynomial bases $\phi_j^m = \binom{m}{j} s^j (1-s)^{m-j}$ are better, with Lebesgue constant 1. But 2 is good.

I think that the proof is very pretty.

One first proves by a contour integral that

$$L_{m,m}(s) - L_{m-1,m-1}(s) = \frac{1}{m+1}\binom{2m}{m} s^m (1-s)^m \,. \tag{3}$$

Then the Lebesgue function can be written

$$L_{m,m}(s) = \sum_{j=0}^{m} \frac{1}{j+1}\binom{2j}{j} s^j (1-s)^j \tag{4}$$

and this is, with $x = s(1-s)$, a truncation of the ordinary generating function for **Catalan numbers**. It is maximal when $s = 1/2$ and bounded above by 2.

Then as was illustrated in Figure 6 we can translate this result to the interval $-1 \le t \le 1$ for “fair” comparison to other bases. Details in the Maple Transactions paper.

The picture I want to get to *

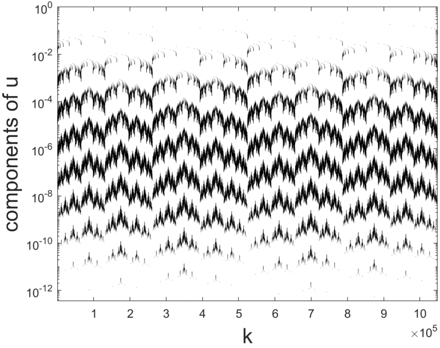

Figure 8: A Fractal Eigenvector

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* Narrator: He's not going to get to this picture in 25 minutes. You will have to read the paper.

Thank you!

Happy to take questions!

This work was partially supported by NSERC grant RGPIN-2020-06438, and partially supported by the grant PID2020-113192GB-I00 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish MICINN, and both the Fields and Heilbronn Institutes for Mathematical Research. I also thank my co-authors, all of them everywhere.

Finally I thank the organizers and the IHP for this invitation.